# Examples of 3D graph sketching

**Yue Kwok Choy**

Visualizing a 3D graph is interesting. The following examples give you some hints in sketching.

1. Sketch the graph $z = x^2 + y^2 - 4x + 6y + 13$

$z = x^2 + y^2 - 4x + 6y + 13 = (x-2)^2 + (y+3)^2$

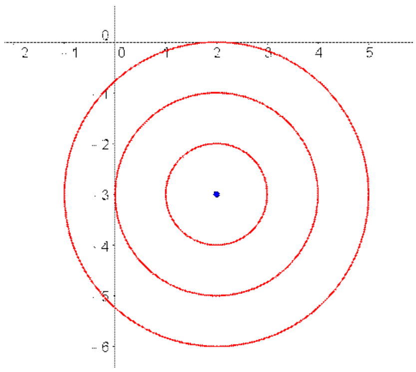
(A) x-y plane view

When $z = 0$, $(x-2)^2 + (y+3)^2 = 0$, we get a point $(2, -3)$.

When $z = 1$, $(x-2)^2 + (y+3)^2 = 1$, we get a circle with centre $(2, -3)$ and radius 1.

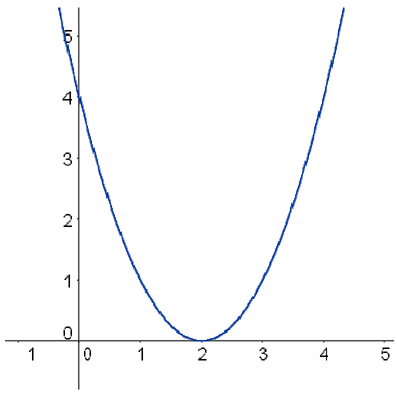
When $z = 4$, $(x-2)^2 + (y+3)^2 = 2^2$, we get a circle with centre $(2, -3)$ and radius 2.

When $z = 9$, $(x-2)^2 + (y+3)^2 = 3^2$, we get a circle with centre $(2, -3)$ and radius 3.



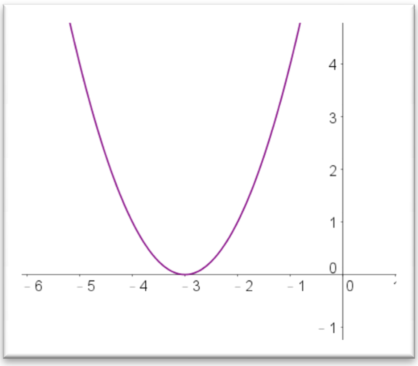
(B) x-z plane view

When $y = -3$, we get $z = (x-2)^2$, which is a parabola.

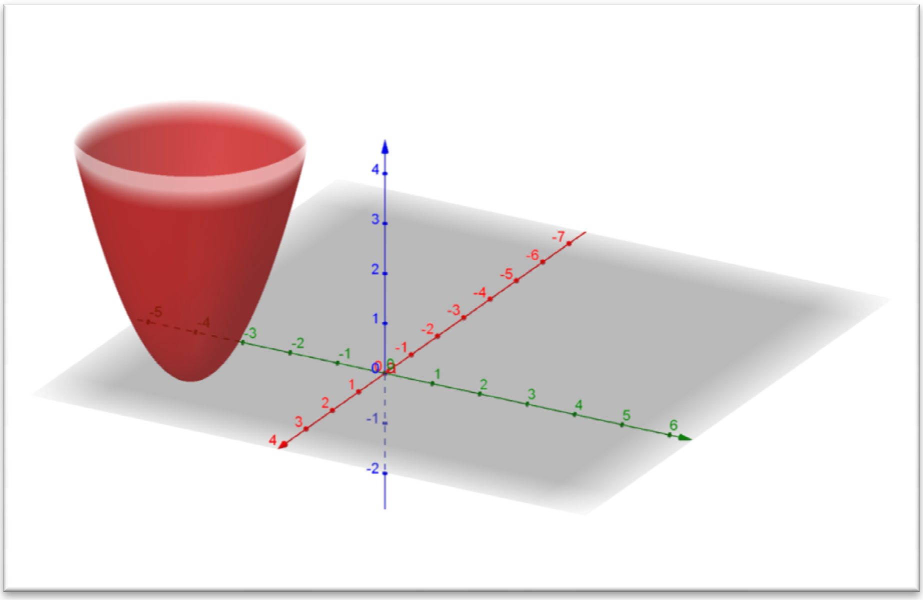


(C) y-z plane view

When $x = 2$, we get $z = (y+3)^2$, which is a parabola.



The result is a **paraboloid**, shown below:

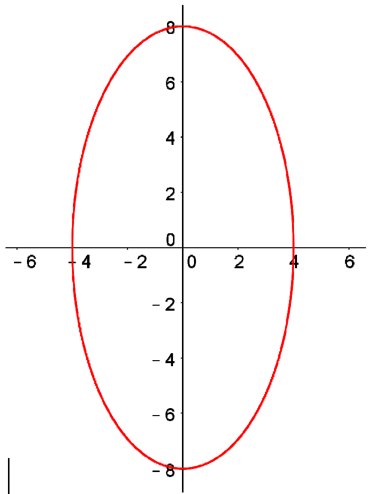



2. Sketch the graph $z = \sqrt{64 - 4x^2 - y^2}$

(A) x-y plane view

When $z = 0$, $64 - 4x^2 - y^2 = 0$,

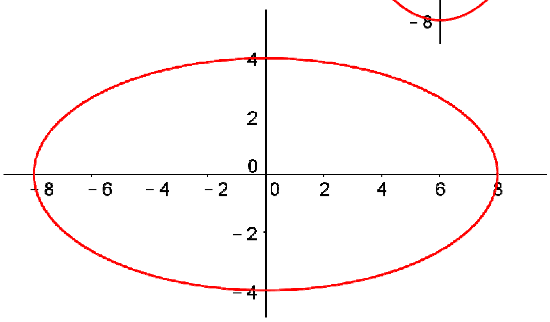
$$\frac{x^2}{4^2} + \frac{y^2}{8^2} = 1, \text{ which is an ellipse.}$$




(B) x-z plane view

When $y = 0$, $z^2 = 64 - 4x^2$,

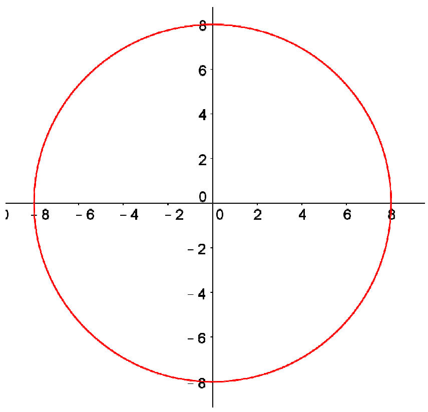
$$\frac{x^2}{8^2} + \frac{z^2}{4^2} = 1, \text{ which is an ellipse.}$$




(C) y-z plane view

When $x = 0$, $z^2 = 64 - y^2$

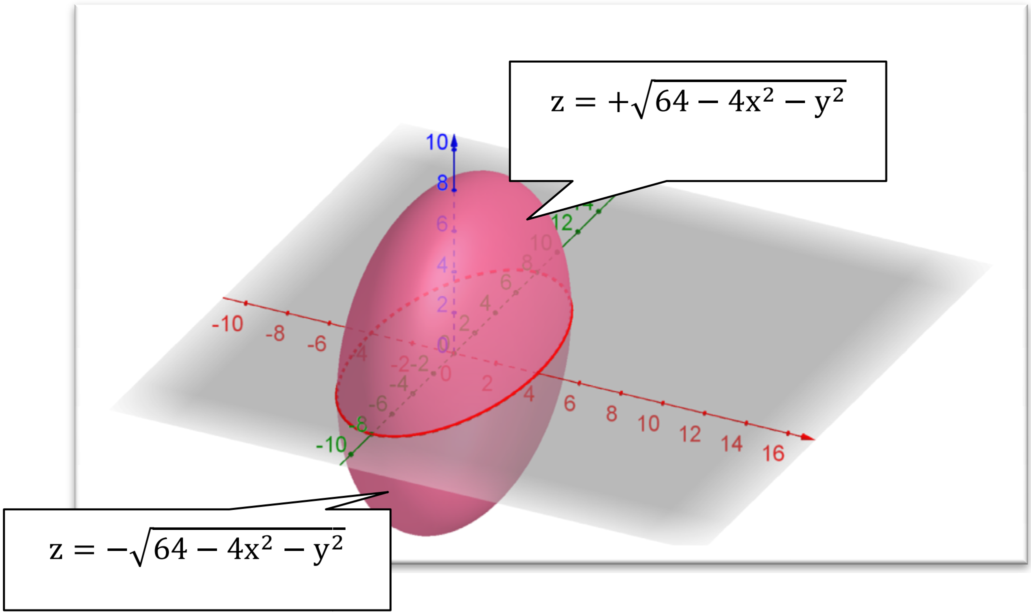
$y^2 + z^2 = 8^2$, which is a circle of radius 8.



The graph of $4x^2 + y^2 + z^2 = 64$ is composed of two parts:

$z = +\sqrt{64 - 4x^2 - y^2}$ and $z = -\sqrt{64 - 4x^2 - y^2}$

The given graph is a **top half of ellipsoid**, shown below:




3. Sketch the graph $-\frac{x^2}{4} - \frac{y^2}{9} + \frac{z^2}{16} = 1$

(A) x-y plane view

When $z = 0$, $-\frac{x^2}{4} - \frac{y^2}{9} = 1$, which is an empty set for $(x, y)$.

When $z = \pm 4$, $-\frac{x^2}{4} - \frac{y^2}{9} = 0$ , we get two points $(0,0,4), (0,0,-4)$

When $z > 4 \; or \; z < -4$, take $z = k$, $\frac{x^2}{4} + \frac{y^2}{9} = \frac{k^2}{16} - 1$, $\frac{x^2}{\frac{4}{\frac{k^2}{16}-1}} + \frac{y^2}{\frac{9}{\frac{k^2}{16}-1}} = 1$, which is an ellipse.

(B) x-z plane view

When $y = 0$ (or any constant k),

$-\frac{x^2}{4} + \frac{z^2}{16} = 1$, $\frac{z^2}{16} - \frac{x^2}{4} = 1$,

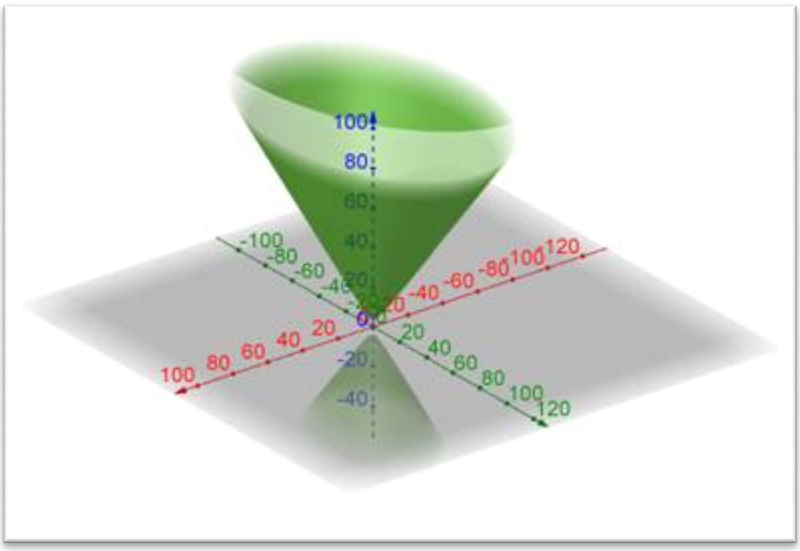
which is a hyperbola.

(C) y-z plane view

When $x = 0$ (or any constant k),

$-\frac{y^2}{9} + \frac{z^2}{16} = 1$, $\frac{z^2}{16} - \frac{y^2}{9} = 1$,

which is a hyperbola.



The graph, which is a **two sheeted- hyperboloid**, is shown on the left.